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Date: 07.12.2002
Homework number: 3
Homework Title: Exercise 4.1

Problem description:

(a) Prove that 5 is an eigenvalue of the matrix

$$A = \begin{bmatrix} 6 & 3 & 3 & 1 \\ 0 & 7 & 4 & 5 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 8 \end{bmatrix}.$$

(b) Exhibit an eigenvector of A corresponding to the eigenvalue 5.

Problem solution:

(a) Trivial.

(b) Solving $Ax = \lambda x$ yields eigenvector corresponding to λ.

Results:

(a) Since A is a upper-triangular matrix, the diagonal contains all eigenvalues of A. Second possibility if A were not triangular: $det(A - \lambda I) = 0$

(b) Solving linear system

$$\begin{aligned} 6x_1 + 3x_2 + 3x_3 + x_4 &= 5x_1 \\ 7x_2 + 4x_3 + 5x_4 &= 5x_2 \\ 5x_3 + 4x_4 &= 5x_3 \\ 8x_4 &= 5x_4 \end{aligned}$$

using backward substitution: $x_4 = 0 \Rightarrow x_3 = 1 \Rightarrow x_2 = -2 \Rightarrow x_1 = 3$

Discussion and Comments: