

First name: Harald
Last name: Röck
Date: 18. Dezember 2002
Homework number: 4
Homework Title: Exercise 4.3

Problem description:

let

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}.$$

- (a) What is the characteristic polynomial of \mathbf{A} ?
- (b) What are the roots of the characteristic polynomial of \mathbf{A} ?
- (c) What are the eigenvalues of \mathbf{A} ?
- (d) What are the eigenvectors of \mathbf{A} ?

Problem solution:

(a)

$$\det \begin{bmatrix} 1-r & 4 \\ 1 & 1-r \end{bmatrix} = 0$$

$$\implies (1-r)(1-r) - 4 = 0 \implies r^2 - 2r - 3 = 0$$

(b)

$$\implies r_{1,2} = 1 \mp \sqrt{1+3} = 1 \mp 2$$

$$r_1 = 3; r_2 = -1$$

(c)

$$e_1 = 3; e_2 = -1$$

(d)

$$\mathbf{A}x = 3x$$

$$x_1 + 4x_2 = 3x_1$$

$$x_1 + x_2 = 3x_2$$

$$\rightarrow -x_1 + 2x_2 = 0$$

$$\implies x = [2 \ 1]^T = v_1$$

$$\mathbf{A}x = -x$$

$$x_1 + 4x_2 = -x_1$$

$$x_1 + x_2 = -x_2$$

$$\rightarrow x_1 + 2x_2 = 0$$

$$\implies x = [2 \ -1]^T = v_2$$