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Scientific Computing - Homeworks

4.3 Let $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix}$

Your answers to the following questions should be numeric and specific to this particular matrix , not just the general definitions .

e) Perform one iteration of power iteration on A , using $X_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ as starting vector .

$$X_1 = A X_0 = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 5 \\ 2 \end{bmatrix} .$$

f) To what eigenvector of A will power iteration ultimately converge ?

The power iteration will converge to dominant eigenvalue λ_2 (with maximum modulus) with the corresponding eigenvector V_2 :

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} . \quad A - \lambda E = 0 \quad \Rightarrow$$

$$\lambda_1 = -1 \quad V_1 = \begin{bmatrix} -1 & 0.5 \end{bmatrix}$$

$$\lambda_2 = 3 \quad V_2 = \begin{bmatrix} 1 & 0.5 \end{bmatrix}$$

The power iteration will converge to $V_2 = \begin{bmatrix} 1 & 0.5 \end{bmatrix}$.

g) What eigenvalue estimate is given by the Rayleigh quotient, using the vector $X = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$?

$$X \lambda = A X = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\Rightarrow \lambda = X^T A X / X^T X = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \begin{bmatrix} 5 \\ 2 \end{bmatrix} / \begin{bmatrix} 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 7 / 2 = 3.5$$

\Rightarrow after n-Iterations :

$\Rightarrow \lambda \sim 3$. Rayleigh quotient give an estimation of the dominant eigenvalue .