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Homework Title: Exercise 3.24

Problem description:

Verify that the dominant term in the operation count (number of multiplcations or number of additions) for solving an $m \times n$ linear least squares problem using the normal equations and Cholesky factorization in $mn^2/6 + n^3/6$.

Problem solution:

Calculate number of operations for

(a) Normal equations

(b) Cholesky factorization

Results:

(a) $A^T A$: The resulting matrix is symmetric has $n \times n$ elements. Therefore actually $\frac{n(n+1)}{2}$ elements have to be calculated. For each element m multiplications and $m-1$ additions have to be made.
$A^T b$: The resulting vector has n elements, each requiring m multiplications and $m-1$ additions.
If additions are left out, because the computing-costs are much less than for multiplications and assuming a large n, the dominant term for normal equations is

$$\frac{n^2 m}{2}$$

(b) Cholesky factorization needs $\sum_{j=1}^{n} \sum_{k=1}^{j-1} (n-j+1)$ multiplications and the same number of additions plus n calculations of a square root and $\sum_{j=1}^{n} n-j$ divisions.

$$\sum_{j=1}^{n} \sum_{k=1}^{j-1} (n-j+1) =$$

$$\sum_{j=1}^{n} (j-1)n - (j-1)j + (j-1) =$$

$$\sum_{j=1}^{n} jn - n - j^2 + j + j - 1 =$$

$$n\frac{n(n+1)}{2} - n^2 - \frac{n(n+1)(2n+1)}{6} + 2\frac{n(n+1)}{2} - n =$$

$$\frac{3(n+2)n(n+1)}{6} - \frac{(2n+1)n(n+1)}{6} - n^2 - n =$$

$$\frac{(n+5)n(n+1)}{6} - n^2 - n =$$

$$\frac{n^3 + 5n^2 + n^2 + 5n - 6n^2 - 6n}{6} =$$

$$\frac{n^3 - n}{6}$$

Thus the dominant term for cholesky factorization is (again leaving out additions, as well as n square roots on $O(n^2)$ divisions)

$$\frac{n^3}{6}$$

The overall dominant term in the operation count evaluates to

$$\frac{n^2 m}{2} + \frac{n^3}{6}$$

Discussion and Comments: