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### Scientific Computing - Homework

**3.4 In fitting a straight line  $y = x_0 + x_1 t$  to the three data points  $(t_i, y_i) = (0,0), (1,0), (1,1)$ , is the least squares solution unique? Why?**

Fitting the straight line  $y$  with the three data points gives :

$$A x = [ 1 \ t_1 ; 1 \ t_2 ; 1 \ t_3 ] [ x_0 \ x_1 ]^T \sim [ y_1 ; y_2 ; y_3 ] = b$$

This is an overdetermined  $3 \times 2$  linear system :

$$A x = [ 1 \ 0 ; 1 \ 1 ; 1 \ 1 ] [ x_0 \ x_1 ]^T \sim [ 0 ; 0 ; 1 ] = b$$

The solution is unique if , and only if , the columns of  $A$  are linearly independent .

In this case  $A$  is  $3 \times 2$  with columns :  $(1 \ 1 \ 1)$  and  $(0 \ 1 \ 1)$  who are linearly independent ,  
 $\text{rank}(A) = 2$  ,

and the least squares solution is unique .