First name: Christoph
Last Name: Oberauer
Email: coberau@cosy.sbg.ac.at
Date: 15.11.2002
Homework number: 1
Homework Title: Exercise 1.10

Problem description:
Consider the expression:

$$\frac{1}{1-x} - \frac{1}{1+x}$$

assuming $x \neq \pm 1$.

(a) For what range of values of x it is difficult to compute this expression accurately to floating point arithmetic?

(b) Give a rearrangement of the terms such that for the range of x in part a, the computation is more accurate in floating point arithmetic.

Problem solution: Define ϵ dependent on machine precession (taking into account the rounding rule), where $fl(1+\epsilon) > 1$.

(a) Try different values for x

(b) Rearrange expression

Results:

(a) This conditions apply:

1. For absolute values of x smaller than ϵ both operands evaluate to 1 due to cancellation, resulting in zero after subtraction.
2. For values of x from the intervals $(1-\epsilon, 1+\epsilon)$ resp. $(-1-\epsilon, -1+\epsilon)$ division by zero occurs.
3. For large absolute values of x $fl(x-1)$ and $fl(x+1)$ (resp. $\frac{1}{x-1}$ and $\frac{1}{x+1}$) become almost equal so accuracy will be lost due to cancellation.
4. Depending on properties of the floating-point system, large absolute values of x can result in underflow in $\frac{1}{1 \pm x}$.

(b) Transform equation to $\frac{2x}{1-x^2}$:

1. $(1-\epsilon, 1+\epsilon)$ and $(-1-\epsilon, -1+\epsilon)$ are still critical intervals.
2. Critical interval for cancellation in denominator around 0 spreads to $(-\sqrt{\epsilon}, \sqrt{\epsilon})$.
   Critical interval for cancellation in enumerator around 0 narrows to $(-\frac{\epsilon}{2}, \frac{\epsilon}{2})$.
3. Maximum absolute value for x limited by $\sqrt{OFL}$ due to x^2.
4. Large values for x not affected by cancellation, but some accuracy lost due to chopping/rounding after multiplication/division.

Discussion and Comments: