

Alin Giurginca
9324156

Scientific Computing - Homeworks

1.17 Let x be a given nonzero floating-point number in a normalized system, and let y be an adjacent floating-point number, also nonzero.

a) What is the minimum possible spacing between x and y ?

Let radix = β , precision = p ($p-1$ bits mantissa, 1 bit for sign), exponent range = $[L, U]$.

The minimum possible spacing between x and y is for the smallest positive number :

$$x = 1.0 * \beta^L$$

and his neighbouring number :

$$y = (1.0 + 2^{-(p-1)}) * \beta^L \quad (-(p-1), \text{ because of 1-bit sign }).$$

so it is :

$$\min = y - x = (1.0 + 2^{-(p-1)}) * \beta^L - 1.0 * \beta^L = 2^{-(p-1)} * \beta^L .$$

b) What is the maximum possible spacing between x and y ?

The maximum possible spacing between x and y is for the biggest positive number :

$$y = 1.0 * \beta^U$$

and his neighbouring number :

$$x = (1.0 - 2^{-(p-1)}) * \beta^U \quad (-(p-1), \text{ because of 1-bit sign }).$$

so it is :

$$\max = x - y = 1.0 * \beta^U - (1.0 - 2^{-(p-1)}) * \beta^U = 2^{-(p-1)} * \beta^U .$$