REVIEW QUESTIONS - CHAPTER 8

8.1. True or false: Evaluating a definite integral is always a well-conditioned problem.
8.2. True or false: Because it is based on polynomial interpolation of degree one higher, the trapezoid rule is generally more accurate than the midpoint rule.
8.3. True or false: The degree of a quadrature rule is the degree of the interpolating polynomial on which the rule is based.
8.4. True or false: An n-point Newton-Cotes quadrature rule is always of degree n - 1.
8.5. True or false: Gaussian quadrature rules of different orders never have any points in common.
8.6. What conditions are both necessary and sufficient for a Riemann integral to exist?
8.7. (a) Under what conditions is a definite integral likely to be sensitive to small perturbations in the integrand?
(b) Under what conditions is a definite integral likely to be sensitive to small perturbations in the limits of integration?
8.8. What is the difference between an open quadrature rule and a closed quadrature rule?
8.9. Name two different methods for computing the weights corresponding to a given set of nodes of a quadrature rule.
8.10. How can you estimate the error in a quadrature rule without computing the derivatives of the integrand function that would be required by a Taylor series expansion?
8.11. (a) How does the node placement differ between Newton-Cotes quadrature and Clenshaw-Curtis quadrature?
(b) Which would you expect to be more accurate for the same number of nodes? Why?
8.12. (a) How does the node placement differ between Newton-Cotes quadrature and Gaussian quadrature?
8.13. (a) If a quadrature rule for an interval [a, b] is based on polynomial interpolation at n equally spaced points in the interval, what is the highest degree such that the rule integrates all polynomials of that degree exactly?
(b) How would your answer change if the points were optimally placed to integrate the highest possible degree polynomials exactly?
8.14. (a) Would you expect an n-point Newton-Cotes quadrature rule to work well for integrating Runge's function, $\int_{-1}^{1}(1+25x^2)^{-1}\,dx$ if n is very large? Why?
(b) Would you expect an n-point Chenshaw-Curtis quadrature rule to work well for integrating Runge's function, $\int_{-1}^{1}(1+25x^2)^{-1}\,dx$ if n is very large? Why?
8.15. (a) What is the degree of Simpson's rule for numerical quadrature?
(b) What is the degree of an n-point Gaussian quadrature rule?
8.16. Newton-Cotes and Gaussian quadrature rules are both based on polynomial interpolation.
(a) What specific property characterizes a Newton-Cotes quadrature rule for a given number of nodes?
(b) What specific property characterizes a Gaussian quadrature rule for a given number of nodes?
8.17. (a) Explain how the midpoint rule, which is based on interpolation by a polynomial of degree zero, can nevertheless integrate polynomials of degree one exactly.
(b) Is the midpoint rule a Gaussian quadrature rule? Explain your answer.
8.18. Suppose that the quadrature rule $\int_a^b$ f(x) dx ≈ sum_(i=1..n) w_if(x_i) is exact for all constant functions. What does this imply about the weights w_i or the nodes x_i ?
8.19. Why is it important for all of the weights of a quadrature rule to be nonnegative?

8.20. If the integrand has an integrable singularity at one endpoint of the interval of integration, which type of quadrature rule would be better to use, a closed Newton-Cotes rule or a Gaussian rule? Why?
8.21. What is the degree of each of the following types of numerical quadrature rules?
(a) An *n*-point Newton-Cotes rule, where *n* is odd
(b) An *n*-point Newton-Cotes rule, where *n* is even
(c) An *n*-point Gaussian rule
(d) What accounts for the difference between the answers to parts *a* and *b*?
(e) What accounts for the difference between the answers to parts *b* and *c*?
8.22. For each of the following properties, state which type of quadrature, Newton-Cotes or Gaussian, more accurately fits the description:
(a) Easier to compute nodes and weights
(b) Easier to apply for a general interval [*a*, *b*]
(c) More accurate for the same number of nodes
(d) Has maximal degree for the number of nodes
(e) Nodes easy to reuse as order of rule changes
8.23. What is the relationship between Gaussian quadrature and orthogonal polynomials?
8.24. (a) What does it meant for a sequence of quadrature rules to be progressive?
(b) Why is this property important?
8.25. (a) What is the advantage of using a Gauss-Kronrod pair of quadrature rules, such as G_7 and K_{15}, compared with using two Gaussian rules, such as G_7 and G_{15}, to obtain an approximate integral with error estimate?
(b) How many evaluations of the integrand function are required to evaluate *both* of the rules G_7 and K_{15} in a given interval?
8.26. Rank the following types of quadrature rules in order of their degree for the same number of nodes (1 for highest degree, etc.):
(a) Newton-Cotes (b) Gaussian (c) Kronrod
8.27. (a) What is a composite quadrature rule?
(b) Why is a composite quadrature rule preferable to an ordinary quadrature rule for achieving high accuracy in numerically computing a definite integral on a given interval?
(c) In using the composite trapezoid quadrature rule to approximate a definite integral on an interval [*a*, *b*], by what factor is the overall error reduced if the mesh size (i.e.subinterval length) *h* is halved?
8.28. (a) Describe in general terms how adaptive quadrature works.
(b) How can one obtain the error estimate needed?
(c) Under what circumstances might such a procedure produce a result that is seriously in error?
(d) Under what circumstances might such a procedure be very inefficient?
8.29. What is the most efficient way to use an adaptive quadrature routine for computing a definite integral whose integrand has a known discontinuity within the interval of integration?
8.30. What is a good way to integrate tabular data (i.e., an integrand whose value is known only at a discrete set of points)?
8.31. (a) How might one use a standard quadrature routine, designed for integrating over a finite interval, to integrate a function over an unbounded interval?
(b) What precautions would need to be taken to ensure a good result?
8.32. How might one use a standard one-dimensional quadrature routine to compute the value of a double integral over a rectangular region?
8.33. Why is Monte Carlo *not* a practical method for computing one-dimensional integrals?

8.34. Relative to other methods for numerical quadrature, why is the Monte Carlo method more effective in higher dimensions than in low dimensions?
8.35. Explain why integral equations of the first kind with smooth kernels are always ill-conditioned.
8.36. Explain how a quadrature rule can be used to solve an integral equation numerically. What type of computational problem results?
8.37. In solving an integral equation of the first kind by numerical quadrature, does the solution always improve if the order of the quadrature rule is increased or the mesh size is decreased? Why?
8.38. List three approaches for obtaining a meaningful solution to an ill-conditioned linear system approximating an integral equation of the first kind.
8.39. Consider the problem of approximating the derivative of a function that is measured or sampled at only a finite number of points.
(a) One way to obtain an approximate derivative is to interpolate the discrete data points and then differentiate the interpolant. Is this a good method for approximating the derivative? Why?
(b) Similarly, one can approximate the integral of a function given by such discrete data by integrating the interpolant. Is this a good method for computing the integral? Why?
8.40. Comparing integration and differentiation, which problem is inherently better conditioned? Why?
8.41. (a) Suggest a good method for numerically approximating the derivative of a function whose value is given only at a discrete set of data points.
(b) For this problem, what would be the effect of noisy data, and how would you cope with it in your numerical method?
8.42. List two methods for deriving finite difference approximations to the derivatives of a given function.
8.43. Briefly describe the basic idea of automatic differentiation. What basic result from calculus does it depend heavily on?
8.44. (a) Explain the basic idea of Richardson extrapolation.
(b) Does it give a more accurate answer than the values on which it is based?
(c) Does extrapolation to step size zero mean that the result is exact (i.e., the error is zero)?
8.45. What is meant by Romberg integration?