REVIEW QUESTIONS - CHAPTER 4

4.1. True or false: The eigenvalues of a matrix are not necessarily all distinct.
4.2. True or false: All the eigenvalues of a real matrix are necessarily real.
4.3. True or false: An eigenvector corresponding to a given eigenvalue of a matrix is unique.
4.4. True or false: Every $n \times n$ matrix A has n linearly independent eigenvectors.
4.5. True or false: If an $n \times n$ matrix is singular, then it does not have a full set of n linearly independent eigenvectors.
4.6. True or false: A square matrix A is singular if, and only if, 0 is one of its eigenvalues.
4.7. True or false: If $\lambda = 0$ for every eigenvalue λ of a matrix A, then $A = 0$.
4.8. True or false: The diagonal elements of a complex Hermitian matrix must be real.
4.9. True or false: The eigenvalues of a complex Hermitian matrix must be real.
4.10. True or false: If two matrices have the same eigenvalues, then the two matrices are similar.
4.11. True or false: If two matrices are similar, then they have the same eigenvectors.
4.12. True or false: Given any arbitrary square matrix, there is some diagonal matrix that is similar to it.
4.13. True or false: Given any arbitrary square matrix, there is some triangular matrix that is unitarily similar to it.
4.14. True or false: The condition number of a matrix that determines the sensitivity of the solution to a system of linear equations also determines the sensitivity of the eigenvalues and eigenvectors to perturbations in the matrix.
4.15. True or false: The eigenvalues of a real symmetric or complex Hermitian matrix are always well-conditioned.
4.16. True or false: A matrix that is both symmetric and Hessenberg must be tridiagonal.
4.17. True or false: If an $n \times n$ matrix A has distinct eigenvalues, then QR iteration applied to A necessarily converges to a diagonal matrix.
4.18. True or false: For a square matrix, the eigenvalues and the singular values are the same.
4.19. Explain the distinction between a right eigenvector and a left eigenvector.
4.20. What is meant by the spectral radius of a matrix?
4.21. For a given matrix A,
(a) Can the same eigenvalue correspond to two different eigenvectors?
(b) Can the same eigenvector correspond to two different eigenvalues?
4.22. What is meant by the characteristic polynomial of a matrix? What does it have to do with eigenvalues?
4.23. Explain the distinction between algebraic multiplicity and geometric multiplicity of an eigenvalue.
4.24. What is meant by an invariant subspace for a given matrix A?
4.25. What are the eigenvalues and eigenvectors of a diagonal matrix? Give an example.
4.26. Which of the following conditions necessarily imply that an $n \times n$ real matrix A is diagonalizable (i.e., is similar to a diagonal matrix)?
(a) A has n distinct eigenvalues. (b) A has only real eigenvalues. (c) A is nonsingular.
(d) A is equal to its transpose. (e) A commutes with its transpose.
4.27. Which of the following classes of matrices necessarily have all real eigenvalues?
(a) Real symmetric (b) Real triangular (c) Arbitrary real (d) Complex symmetric (e) Complex Hermitian (f) Complex triangular with real diagonal (g) Arbitrary complex
4.28. Let A and B be similar matrices, i.e., $B = T^{-1} AT$ for some nonsingular matrix T. If y is an eigenvector of B, then exhibit an eigenvector of A.

4.29. Give an example of a matrix that is not diagonalizable, i.e., that is not similar to any diagonal matrix.
4.30. The eigenvalues of a matrix are the roots of its characteristic polynomial. Does this fact provide a generally effective numerical method for computing the eigenvalues? Why?
4.31. Before applying QR iteration to find the eigenvalues of a matrix, the matrix is usually first transformed to a simpler form. For each type of matrix listed below, what intermediate form is appropriate?
(a) A general real matrix (b) A real symmetric matrix
4.32. A general matrix can be reduced to triangular form by a single QR factorization, and the eigenvalues of a triangular matrix are its diagonal entries. Does this procedure suffice to compute the eigenvalues of the original matrix? Why?
4.33. Gauss-Jordan elimination reduces a matrix to diagonal form. Does this make the eigenvalues of the matrix obvious? Why?
4.34. (a) Why is the Jacobi method for computing all the eigenvalues of a real symmetric matrix relatively slowly convergent?
(b) Name a method that is faster, and explain briefly why it is faster.
4.35. For which of the following classes of matrices of order n can the eigenvalues be computed in a finite number of steps for arbitrary n?
(a) Diagonal (b) Tridiagonal (c) Triangular (d) Hessenberg (e) General real matrix with distinct eigenvalues (f) General real matrix with eigenvalues that are not necessarily distinct
4.36. In using QR iteration for computing the eigenvalues of a matrix, why is the matrix usually first reduced to some simpler form, such as Hessenberg or tridiagonal?
4.37. Applied to a given matrix A, QR iteration for computing eigenvalues converges to either diagonal or triangular form. What property of A determines which of these two forms is obtained?
4.38. As a preliminary step before computing its eigenvalues, a matrix A is often first reduced to Hessenberg form by a unitary similarity transformation. Why stop there? If such a preliminary reduction to Hessenberg form is good, wouldn't triangular form be even better? What is wrong with this argument?
4.39. If you had a routine for computing all the eigenvalues of a nonsymmetric matrix, how could you use it to compute the roots of any polynomial?
4.40. Order the following algorithms 1 through 4, from least work required to most work required, for a general square matrix A:
(a) LU factorization by Gaussian elimination with partial pivoting
(b) Computing all of the eigenvalues and eigenvectors
(c) Solving an upper triangular system by backsubstitution
(d) Computing the inverse of the matrix
4.41. Power iteration converges to which eigenvector of a matrix?
4.42. (a) If a matrix A has a simple dominant eigenvalue λ_1, what quantity determines the convergence rate of the power method for computing λ_1?
(b) How can the convergence rate of power iteration be improved?
4.43. Given an approximate eigenvector x for a matrix A, what is the best estimate (in the least squares sense) for the corresponding eigenvalue?
4.44. List three conditions under which power iteration for computing an eigenvalue may fail.
4.45. Inverse iteration converges to which eigenvector of a matrix?
4.46. In power iteration or inverse iteration, why are the vector iterates normalized at each iteration?
4.47. What is the main reason that shifts are used in iterative methods for computing eigenvalues, such as the power, inverse iteration, and QR iteration methods?

4.48. Given a general square matrix A, what method would you use to compute the following?
(a) The smallest eigenvalue of A (b) The largest eigenvalue of A (c) The eigenvalue of A closest to some specified scalar β (d) All of the eigenvalues of A
4.49. (a) Given an approximate eigenvalue λ for a matrix, how can one obtain a good approximate eigenvector?
(b) Given an approximate eigenvector x for a matrix, how can one obtain a good approximate eigenvalue?
4.50. What is a Krylov sequence, and for what purpose is it useful?
4.51. Why is the Lanczos method faster than power iteration for computing a few eigenvalues of a real symmetric matrix?
4.52. What features make the Lanczos method suitable for large sparse symmetric eigenvalue problems?
4.53. What is meant by the *inertia* of a real symmetric matrix?
4.54. (a) What is meant by a *congruence* transformation of a real symmetric matrix?
(b) What properties of the matrix, if any, are preserved by such a transformation?
4.55. Explain briefly how spectrum-slicing methods work for computing individual eigenvalues of a real symmetric matrix.
4.56. (a) List two reasons why converting a generalized eigenvalue problem $Ax = \lambda Bx$ to the standard eigenvalue problem $(B^{-1} A)x = \lambda x$ might not be a good idea.
(b) What is a better approach?
4.57. How are the singular values of an $m \times n$ real matrix A related to the eigenvalues of the $n \times n$ matrix $A^T A$?
4.58. Is forming $A^T A$ and computing its eigenvalues a good way to compute the singular values of a matrix A? Why?