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Homework number: 7
Homework Title: Exercise 8.10

Problem description:

Newton-Cotes quadrature rules are derived by fixing the nodes and then determining the corresponding weights by the method of undetermined coefficients so that the degree is maximized for the given nodes. The opposite approach could also be taken, with the weights fixed and the nodes to be determined. In a Chebyshev quadrature rule, for example, all of the weights are taken to have the same value, w, thereby eliminating n multiplications in evaluating the resulting quadrature rule, since the single weight can be factored out of the summation.

(a) Use the method of undetermined coefficients to determine the nodes and weights for a threepoint Chebyshev quadrature rule on the interval $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(b) What is the degree of the resulting rule?

Problem solution:

(a),(b) Use the method of the undetermined coefficients (notes: Ch. 8, p.6-9), integrate first six monomials (which gives moment equations) and solve the resulting system to obtain the weights and nodes for the rule. At last determine degree of rule (b).

Results:

(a)

- $w_1 + w_2 + w_3 = \int_{-1}^{1} 1dx = 2$
- $w_1 * x_1 + w_2 * x_2 + w_3 * x_3 = \int_{-1}^{1} xdx = 0$
- $w_1 * x_1^2 + w_2 * x_2^2 + w_3 * x_3^2 = \int_{-1}^{1} x^2dx = \frac{2}{3}$
- $w_1 * x_1^3 + w_2 * x_2^3 + w_3 * x_3^3 = \int_{-1}^{1} x^3dx = 0$
- $w_1 * x_1^4 + w_2 * x_2^4 + w_3 * x_3^4 = \int_{-1}^{1} x^4dx = \frac{2}{5}$
- $w_1 * x_1^5 + w_2 * x_2^5 + w_3 * x_3^5 = \int_{-1}^{1} x^5dx = 0$

Because in Chebyshev quadrature rule all weights have the same value, we can conclude from the first line of the upper system, that w_1, w_2, w_3 must all 3 be equal to $\frac{2}{3}$.

We can use the next 3 equations of the system to get values for x_1, x_2, x_3:

$x_1 + x_2 + x_3 = 0$
$x_1^2 + x_2^2 + x_3^2 = 1$
$x_1^3 + x_2^3 + x_3^3 = 0$

To obtain the solution to this system of nonlinear equations we can use the Newton method for n-dimensions (notes: Chap. 5 p.43-44). We devise the Jacobi matrix from the system take an aribtrary starting vector and get the solution after a few iterations.
I took the vector $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ as starting vector and got the solution after 3 iterations.
The solutions are: $x_1 = \sqrt{-\frac{1}{2}}, x_2 = 0, x_3 = \sqrt{\frac{1}{2}}$
Resulting Rule: $C(f) = \frac{2}{3} * (f(-\frac{1}{2} + f(0) + f(\frac{1}{2}))$

(b) degree = 3

Discussion and Comments: