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Homework Number: 7
Homework Title: Exercise 8.1.2

1 Problem Description

(a) Compute the approximate value of the integral $\int_{0.5}^{1.5} x^3\,dx$, first by the midpoint rule and then by the trapezoid rule.

(b) Use the difference between these two results to estimate the error in each of them.

(c) Combine the two results to obtain the Simpson's rule approximation to the integral.

(d) Would you expect the latter to be exact for this problem? Why?

2 Problem Solution

(a) For the midpoint rule we have $M(f) = (b-a) * f(\frac{a+b}{2})$. In this case

$$M(f) = (1.5 - 0.5) * (\frac{1.5 + 0.5}{2})^3 = 1 * 1^3 = 1$$

For the trapezoid rule we have $T(f) = \frac{b-a}{2} * (f(a) + f(b))$. So we get

$$T(f) = \frac{1.5 - 0.5}{2} * (0.5^3 + 1.5^3) = 0.5 * 3.5 = 1.75$$

(b) To estimate the error, we use the formula $E(f) \approx \frac{T(f)-M(f)}{3}$. In this case, we get

$$E(f) \approx \frac{1.75 - 1}{3} = \frac{0.75}{3} = 0.25$$

(c) For the Simpson's rule we use

$$S(f) = \frac{2}{3} * M(f) + \frac{1}{3} * T(f) = \frac{2}{3} * 1 + \frac{1}{3} * 1.75 = 1.25$$

(d) Yes

3 Results

(a) The solution with midpoint rule is 1 and with trapezoid rule it is 1.75. The real solution to this integral would be 1.25.

(b) The error in M(f) is about 0.25 and the error in T(f) is about -0.5.

(c) With this weighted combination of M(f) and T(f) we get the solution 1.25, which is also the real solution.

4 Discussion and Comments