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**Homework title:** Exercise 7.1.2

### Problem description:

Given the three data points  $(-1, 1)$ ,  $(0, 2)$ ,  $(1, 1)$  determine the interpolating polynomial of degree two:

1. Using the monomial basis
2. Using the Lagrange basis
3. Using the Newton basis

Show that the three representations give the same polynomial.

### Problem solution:

I will calculate the polynomials using the three methods and show that the three results are equal.

### Results:

1. Monomial Basis:  
 $Ax = b$

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \dots$$

$$\begin{aligned} x_1 - x_2 + x_3 &= 1 \\ x_1 &= 2 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

$$\Rightarrow x_1 = 2, x_3 = -1, x_2 = 0 \Rightarrow x = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \underline{p_2(t) = 2 - t^2}$$

2. Lagrange Basis:

$$e_j = \prod_{k=1, k \neq j}^n \frac{(t - t_k)}{(t_j - t_k)}, j \in 1 \dots n \quad p_{n-1}(t) = \sum_{k=1}^n y_k \cdot e_k(t)$$

$$p_2(t) = 1 \cdot \frac{t(t-1)}{(-1) \cdot (-2)} + 2 \cdot \frac{(t+1) \cdot (t-1)}{1 \cdot (-1)} + 1 \cdot \frac{(t+1) \cdot t}{2 \cdot 1} = \frac{t^2 - t}{2} - \frac{4(t^2 - 1)}{2} + \frac{t^2 + t}{2} =$$

$$\frac{t^2 - t - 4t^2 + 4 + t^2 + t}{2} = \frac{-2t^2 + 4}{2} = \underline{2 - t^2}$$

### 3. Newton Basis:

$$\pi_j(t) = \prod_{k=1}^{j-1} (t - t_k), j \in 1 \dots n \quad p_{n-1}(t) = \sum_{k=1}^n \pi_k \cdot x_k$$

$$p_{n-1}(t) = x_1 + x_2(t - t_1) + x_3(t - t_1)(t - t_2)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & (t_2 - t_1) & 0 \\ 1 & (t_3 - t_1) & (t_3 - t_1)(t_3 - t_2) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \Rightarrow$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{aligned} x_1 &= 1 \\ x_1 + x_2 &= 2 \\ x_1 + 2x_2 + 2x_3 &= 1 \end{aligned}$$

$$\Rightarrow x_2 = -1, x_3 = -1 \Rightarrow p_{n-1}(t) = 1 + 1(t + 1) - 1(t + 1)(t) = 1 + t + 1 - t^2 - t = \underline{\underline{2 - t^2}}$$

### Discussion and Comments:

The results show that the interpolated polynomial is not dependent from the interpolating method. In all three cases the result is  $2 - t^2$ . The decision whether to use one or the other method lies only in the implementation (dis)advantages of the different methods.