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Homework number: 6
Homework Title: Exercise 7.10

Problem description:

(a) For a given set of data points, $t_1, ..., t_n$, define the function $\pi(t)$ by

$\pi(t) = (t - t_1)(t - t_2)...(t - t_n).$

Show that

$\pi'(t_j) = (t_j - t_1)...(t_j - t_{j-1})(t_j - t_{j+1})...(t_j - t_n).$

(b) Use the result of part a to show that the jth Lagrange basis function can be expressed as

$\ell_j(t) = \frac{\pi(t)}{(t-t_j)*\pi'(t_j)}$

Problem solution:

(a) Just simply use the derivative rule for a product and afterwards set t_j as function value for $\pi'(t)$.

(b) Use the definition of Lagrange basis function (Chap.7 p.17) and the results from (a) to come to the requested solution.

Results:

(a) $\pi(t) = (t - t_1)(t - t_2) \dots (t - t_n)$

$$\pi'(t) = 1 * (t - t_2) \dots (t - t_n) + (t - t_1) * 1 * (t - t_3) \dots (t - t_n) + \dots$$
$$+ (t - t_1)(t - t_2) \dots (t - t_{n-1}) * 1$$

If the parameter t is now equal to t_j all of the terms fall off because in every term we get $t_j - t_j = 0$. Only the j-th term survives because the derivation in the former step anihilated $t - t_j$ and only 1 remained $\Rightarrow$

$$\pi'(t_j) = (t_j - t_1)(t_j - t_2) \dots (t_j - t_{j-1})(t_j - t_{j+1}) \dots (t_j - t_n)$$

(b) $\ell_j(t) = \frac{\prod_{k=1,k\neq j}^{n}(t-t_k)}{\prod_{k=1,k\neq j}^{n}(t_j-t_k)} = \frac{(t-t_1)(t-t_2)...(t-t_{j-1})(t-t_{j+1})...(t-t_n)}{(t_j-t_1)(t_j-t_2)...(t_j-t_{j-1})(t_j-t_{j+1})...(t_j-t_n)} =$

$\frac{(t-t_1)(t-t_2)...(t-t_{j-1})(t-t_{j+1})...(t-t_n)}{\pi'(t_j)}$ (used result from (a))

Now we extend with $\frac{(t-t_j)}{(t-t_j)} \Rightarrow$

$\frac{(t-t_1)(t-t_2)...(t-t_{j-1})(t-t_{j+1})...(t-t_n)}{\pi'(t_j)} * \frac{(t-t_j)}{(t-t_j)}$ and finally get (again with (a))

$\frac{\pi(t)}{(t-t_j)\pi'(t_j)} = \ell_j(t)$

Discussion and Comments: -