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Homework number: 7 Homework
Title: Exercise 8.1

Problem description:

- (a) Compute the approximate value of the integral $\int_0^1 x^3 dx$, first by the midpoint rule and then by the trapezoid rule.
(b) Use the difference between these two results to estimate the error in each of them.
(c) Combine the two results to obtain the Simpson's rule approximation to the integral.
(d) Would you expect the latter to be exact for this problem? Why?

Problem solution:

For $f(x) = x^3$

a) In the interval $[a,b]$ midpoint rule is defined as:

$$M(f) = (b - a) * f\left(\frac{a+b}{2}\right)$$

$$\text{In this case: } (1 - 0) * \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

In the interval $[a,b]$ trapezoid rule is defined as:

$$T(f) = \left(\frac{b-a}{2}\right) * (f(a) + f(b))$$

$$\text{In this case: } \left(\frac{1-0}{2}\right) * (0^3 + 1^3) = \frac{1}{2}$$

b) The difference between these two results is:

$$E(f) = \frac{T(f) - M(f)}{3}$$

$$E_T(f) = -2E$$

$$\text{In this case: } E(f) = \frac{\frac{1}{2} - \frac{1}{8}}{3} = \frac{1}{8}$$

$$E_T(f) = -2 * \frac{1}{8} = -\frac{1}{4}$$

c) The combination of two results is:

$$\text{Simpson's rule: } S(f) = \frac{2}{3} * M(f) + \frac{1}{3} * T(f) = \frac{1}{4}$$

d) Yes, Simpson's rule is expected to be exact, because it is of degree 3.

Conclusion:

a) $M(f) = \frac{1}{8}$ and $T(f) = \frac{1}{2}$

b) $E(f) = \frac{1}{8}$ and $E_T(f) = -\frac{1}{4}$.

c) Simpson's rule: $S(f) = \frac{2}{3} * M(f) + \frac{1}{3} * T(f) = \frac{1}{4}$

d) Yes, because Simpson's rule is of degree 3.