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**Date:** 07.04.2002
**Homework number:** 5
**Homework title:** Exercise 5.7

## Problem description:

The gamma function has the following known values: $\Gamma(0.5) = \sqrt{\pi},\ \Gamma(1) = 1,\ \Gamma(0.75) = \frac{\sqrt{\pi}}{2}$. From these three values, determine the approximate value $x$ for which $\Gamma(x) = 1.5$, using one step of each of the following methods:

1. Quadratic interpolation
2. Inverse quadratic interpolation
3. Linear fractional interpolation

## Problem solution:

$x_0 = 0.5,\ x_1 = 0.75,\ x_2 = 1$
The result asked is a value for $x$ so that $\Gamma(x) = 1.5$. If we subtract 1.5 from the value of $\Gamma(x)$ we receive the problem of root finding for the given points. So the new values of $\Gamma(x)$ for our three points are $\Gamma(x) - 1.5$:

$$\Gamma(0.5) = \sqrt{(\pi)} - \frac{3}{2} = \frac{2\sqrt{(\pi)} - 3}{2}, \qquad \Gamma(0.75) = \frac{\sqrt{\pi}}{2} - \frac{3}{2} = \frac{\sqrt{(\pi)} - 3}{2}, \qquad \Gamma(1) = 1 - \frac{3}{2} = -\frac{1}{2}$$

1. Quadratic interpolation (QI) (also called Muller's method):

Equation of the parabola going through the three data points: $P(x) = a(x - x_2)^2 + b(x - x_2) + c$,
where:

$$c = f(x_2)$$

$$b = \frac{(x_0 - x_2)^2[f(x_1) - f(x_2)] - (x_1 - x_2)^2[f(x_0) - f(x_2)]}{(x_0 - x_2)(x_1 - x_2)(x_0 - x_1)}$$

$$a = \frac{(x_1 - x_2)[f(x_0) - f(x_2)] - (x_0 - x_2)[f(x_1) - f(x_2)]}{(x_0 - x_2)(x_1 - x_2)(x_0 - x_1)}$$

...new approximate solution: $x_3 = x_2 - \frac{2c}{b + sign(b)\sqrt{b^2 - 4ac}}$

2. Inverse quadratic interpolation (IQI):

$$u = \frac{f(x_1)}{f(x_2)},\ v = \frac{f(x_1)}{f(x_0)},\ w = \frac{f(x_0)}{f(x_2)}$$

$$p = v(w(u - w)(x_2 - x_1) - (1 - u)(x_1 - x_0))$$
$$q = (w - 1)(u - 1)(v - 1)$$

...new approximate solution: $x_3 = x_1 + \frac{p}{q}$

3. Linear fractional interpolation (LFI):

$$h = \frac{(x_0 - x_2)(x_1 - x_2)(f(x_0) - f(x_1))f(x_2)}{(x_0 - x_2)(f(x_2) - f(x_1))f(x_0) - (x_1 - x_2)(f(x_2) - f(x_0))f(x_2)}$$

...new approximate solution: $x_3 = x_2 + h$

## Results:

I calculated the three results with help of Matlab©:
After one step of each iteration method the new value for $x_3$ is:

1. QI: $x_3 = 1.1399$
2. IQI: $x_3 = 1.1136$
3. LFI: $x_3 = 0.31667$

## Discussion and Comments:

The result of QI and IQI are very similar to each other, whereas LFI returns a totally different result. I checked the results after enough iteration steps so that the asked result of 1.5 would come out with a higher precision of about $10^{-15}$:

1. QI needed 7 steps to come to a result of precision $10^{-16}$ ($x = 0.5953$).
2. IQI needed 18 steps to come to a result with the same precision but an different $x = 2.6628$.
3. LFI needed 13 steps to come to the same result as QI ($x = 0.5953$).

After knowing the actual result of $x = 0.5953$ or $x = 2.6628$, LFI achieved the nearest result after one step but needed more steps afterwords in order to achieve the same result as QI (which needed half the amount of steps as LFI).