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Homework number: 5
Homework Title: Exercise 5.13

Problem description:

Consider the system of equations

$x_1 - 1 = 0,$
$x_1 * x_2 - 1 = 0.$

For what starting point or points, if any, will Newton's method for solving this system fail? Why?

Problem solution:

We could try the method as described in the slides (Chap. 5 p.43-45) with several different starting vectors, but we will have no success at all. Instead we should use the equation with the spectral radius (Chap. 5 p.42) to see what the convergence rate will be.

Results:

The iteration just converges if $\mathbf{p}(\mathbf{G}(\mathbf{x}^*)) < \mathbf{1}$.

We have

$$f(x) = \begin{bmatrix} x_1 & -1 \\ x_1 * x_2 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ x_2 & 0 \end{bmatrix} = G(x)$$

If we calculate the eigenvalues of $G(x)$ we get $\lambda_1 = 1, \lambda_2 = 0$ independent of what x^* we choose because x_2 falls of when multiplied with the zero. This means our max. λ is 1 and we get $1 < 1$. This leads to the conclusion that we do not have convergence for any possible vector $x^* \Rightarrow$ Newton's Method will fail.

Discussion and Comments:

Newton's Method will fail because the intersection line of the two plane tangents (of the two functions) will never intersect the (x_1, x_2) plane for any arbitrary starting vector (we can see this by sketching the two functions). This further means that we can not achieve a new (approximated) point from the Newton method. The only intersection with (x_1, x_2) is given with the vector $\begin{bmatrix} 1 & 1 \end{bmatrix}$ which is the exact solution to the system.