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Homework title: Exercise 4.28

Problem description:

Let $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ be the (real) eigenvalues of an $n \times n$ real symmetric matrix A .

1. To which of the eigenvalues of A is it possible for power iteration to converge by using an appropriately chosen shift s ?
2. In each such case, what value for the shift gives the most rapid convergence?
3. Answer the same two questions for inverse iteration.

Problem solution:

This are questions where a solution need not be calculated in any form or requires much conclusions because they can be found in the notes or books dealing with scientific computing.

Results:

1.
 - Regardless of the value of the shift, power iteration can only converge to an eigenvector corresponding to one of the extreme eigenvalues.
2.
 - With a value of shift either $\frac{\lambda_2 + \lambda_n}{2}$, in which case power iteration converges to a multiple of v_1 , or $\frac{\lambda_1 + \lambda_{n-1}}{2}$, in which case it converges to a multiple of v_n , the best convergence rate is attained. (Where $v_1 \dots v_n$ are the corresponding eigenvectors to the eigenvalues $\lambda_1 \dots \lambda_n$)
3.
 - Without using a shift inverse iteration converges to the smallest eigenvalue of A . In comparison with power iteration, where only the extreme eigenvalues can be calculated, inverse iteration can converge to any eigenvalue when an appropriate shift is chosen!
 - Best convergence rate is given when the chosen shift is very close to an eigenvalue of A .

Discussion and Comments:

Depending on what is needed, power iteration is best for calculating the extreme eigenvalues of A where inverse iteration can be used to obtain the smallest eigenvalue of A . The advantage of inverse iteration is that you can compute any eigenvalue of A when an appropriate shift is chosen where an shift in power iteration can only improve the convergence rate. Inverse iteration is a good method for finding exact eigenvalues when approximate ones have already been found by other means.