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Date: 21.12.01
Homework number: 4
Homework Title: Exercise 4.32

Problem description:

(a) What are the eigenvalues of the Householder transformation

$\mathbf{H} = \mathbf{I} - \mathbf{2} * \frac{\mathbf{v} * \mathbf{v}^\mathbf{T}}{\mathbf{v}^\mathbf{T} * \mathbf{v}}$,

where $\mathbf{v}$ is any nonzero vector?

(b) What are the eigenvalues of the plane rotation

$$G = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$

where $\mathbf{c^2 + s^2 = 1}$?

Problem solution:
The characteristic polynomial says:

$$\lambda \text{ is eigenvalue of matrix A} \Leftrightarrow det(A - \lambda * I) = \begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \dots & & & \dots \\ a_{n1} & \dots & & a_{nn} - \lambda \end{vmatrix} = 0$$

This property is used in (a) and (b) to derive a general solution the the problem.

Results:

(a) If we use the characteristic polynomial with H we get:

$det(H - \lambda * I) = 0$

$\Rightarrow det(I - 2 * \frac{v * v^T}{v^T * v} - \lambda * I) = 0$

$\Rightarrow det(I(1 - \lambda) - 2 * \frac{v * v^T}{v^T * v}) = 0$

We can easily see that $v * v^T$ is a symmetric matrix and $\frac{2}{v^T * v}$ is some scalar $\overline{v}$, which norms the matrix.

$\Rightarrow det(I(1 - \lambda) - \overline{v} * v * v^T) = 0$

If we calculate the eigenvalues of this matrix for any arbitrary vector v (of length n) we get $\lambda_1 = \pm 1, ..., \lambda_n = \pm 1$

(b) If we use the characteristic polynomial with G we get:

$$det(G - \lambda * I) = 0$$

$$\Rightarrow \begin{vmatrix} cos - \lambda & sin \\ -sin & cos - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (cos - \lambda)^2 + sin^2 = 0$$

$$\Rightarrow cos^2 - 2 * \lambda * cos + \lambda^2 + sin^2 = 0$$

$$\Rightarrow \lambda^2 - 2 * \lambda * cos + 1 = 0 \qquad (because\ cos^2 + sin^2 = 1)$$

$$\Rightarrow \lambda_1, \lambda_2 = cos \pm \sqrt{-sin} = cos \pm i * sin$$

So the eigenvalues of the plane rotation are $\lambda_1 = cos + i * sin$ and $\lambda_2 = cos - i * sin$.

Discussion and Comments: -