1 Problem

We consider the Poisson boundary value problem with given frequencies $\boldsymbol{a} = (a_i)_{i=1}^d$ in d dimensions:

$$
\begin{aligned}
\nabla^2 u &= -\pi^2 \left(\sum_{i=1}^d a_i^2 \right) \prod_{i=1}^d \sin(\pi a_i x_i) \quad \text{on } \Omega = [0,1]^d, \\
u|_{\partial\Omega} &= 0,
\end{aligned} \tag{1}
$$

with solution $u(\boldsymbol{x}) = u_a(\boldsymbol{x}) = \prod_{i=1}^d \sin(\pi a_i x_i)$ is being considered.

We obtain the numerical solution u_c using a collocation technique, with $n_c = 3$ nodes and 3 basis functions. The solution u_c is known only in the collocation nodes x_c. We will also use the middle nodes x_m which will in 1D represent the midpoints of the intervals, defined by x_c.

WLS approximation has three parameters in general: the number of neighboring collocation nodes n, the order of basis m and the weight function w. If $n = m + 1$, that the approximation becomes interpolation and the effect of weight disappears.

We obtain the improved version u_i in three ways:

- DIRECT: u_i is computed at the collocation nodes x_c using WLS approximation (with parameters n, m) of u_c. The error indicator is computed simply as $|u_i - u_c|$ and is known at x_c.
- MIDPOINT: u_c is extended to x_m using low order WLS approximation (with parameters n_s, m_s), call the extension $\tilde{u}_c$. u_i is obtained by extending u_c from x_c to x_m as well, but with higher order approximation (with parameters n, m). The error indicator is computed as $|\tilde{u}_c - u_i|$ and is known at x_m.
- MIDPOINT-AND-BACK: u_i is first extended to x_m as low order WLS approximation (with parameters n_s, m_s), call the extension $\tilde{u}_c$. Then u_i is computed at x_c using higher order WLS approximation (with parameters n, m) from $\tilde{u}_c$. The error indicator is computed as $|u_i - u_c|$ and is known at x_c.

This procedure is repeated for each component of ∇u_c as well. The analytical errors shown are computed at the same nodes as the error indicator.

2 1D indicator test

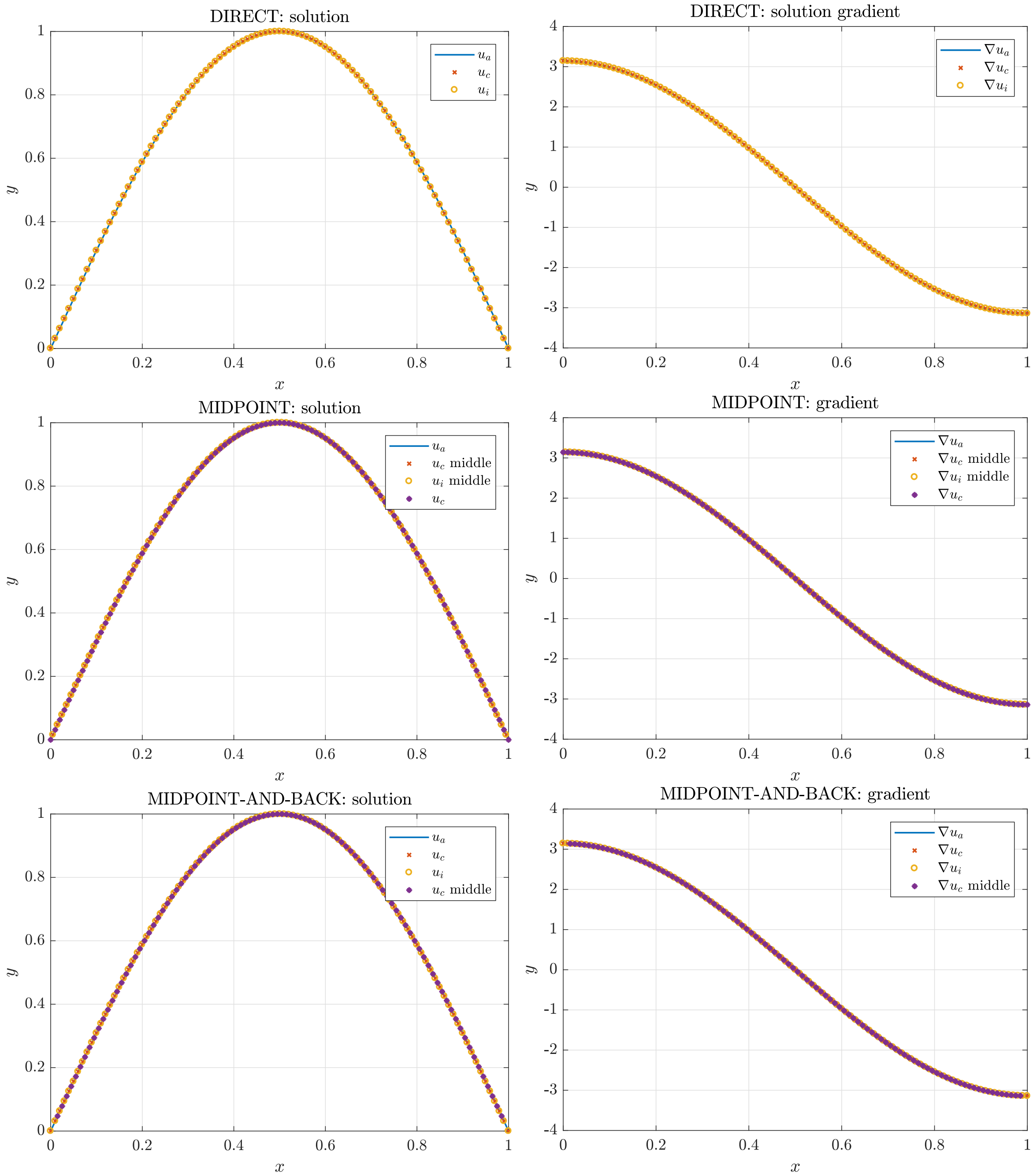

Figure 1: Solution behavior in 1D with $\boldsymbol{a} = (1)$, $n = 5$, $m = 2$ in DIRECT case and $n_s = 2$, $m_s = 1$, $n = 3$, $m = 2$ in other cases.

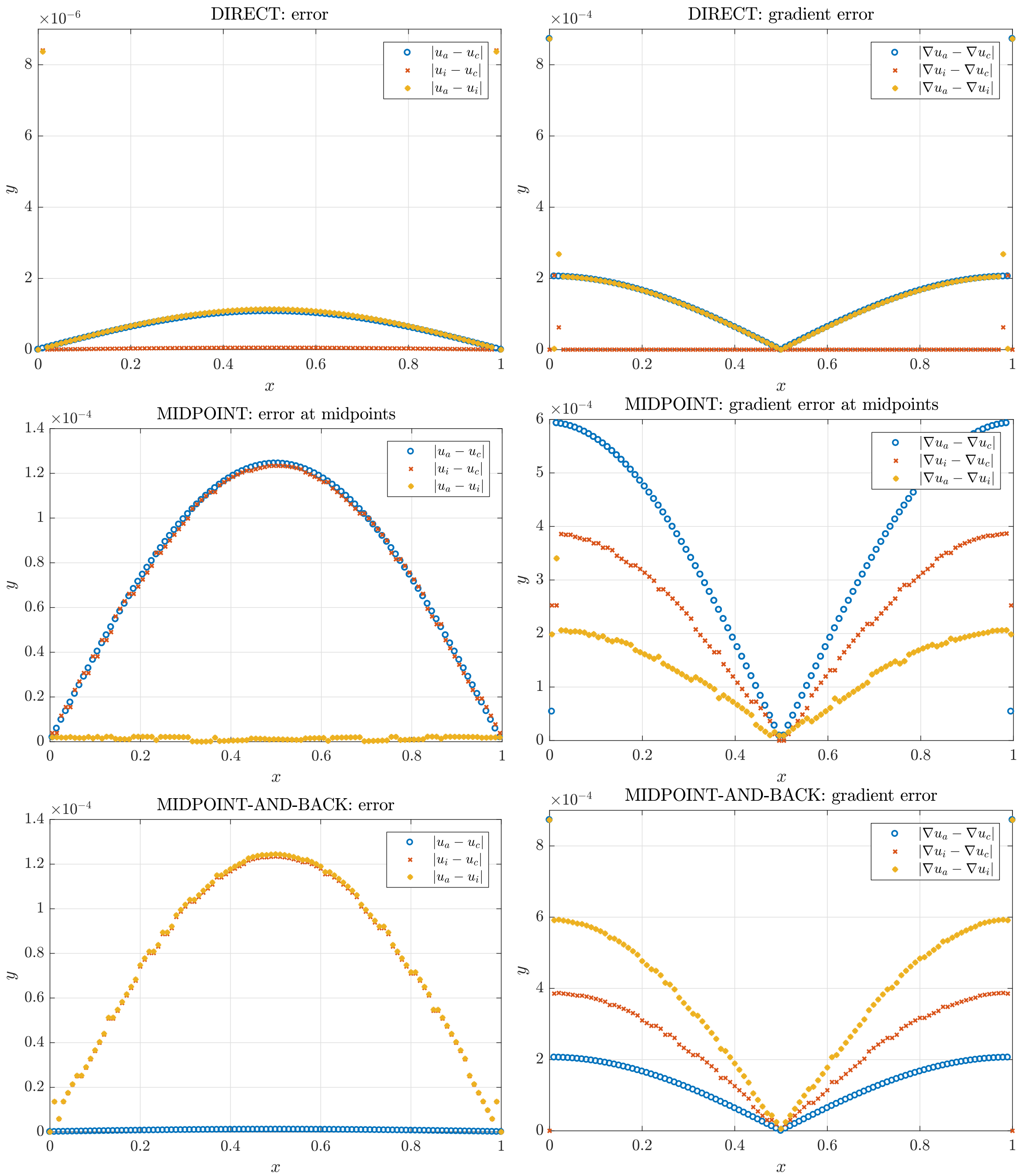

Figure 2: Indicator behavior in 1D with $\boldsymbol{a} = (1)$, $n = 5$, $m = 2$ in DIRECT case and $n_s = 2$, $m_s = 1$, $n = 3$, $m = 2$ in other cases.

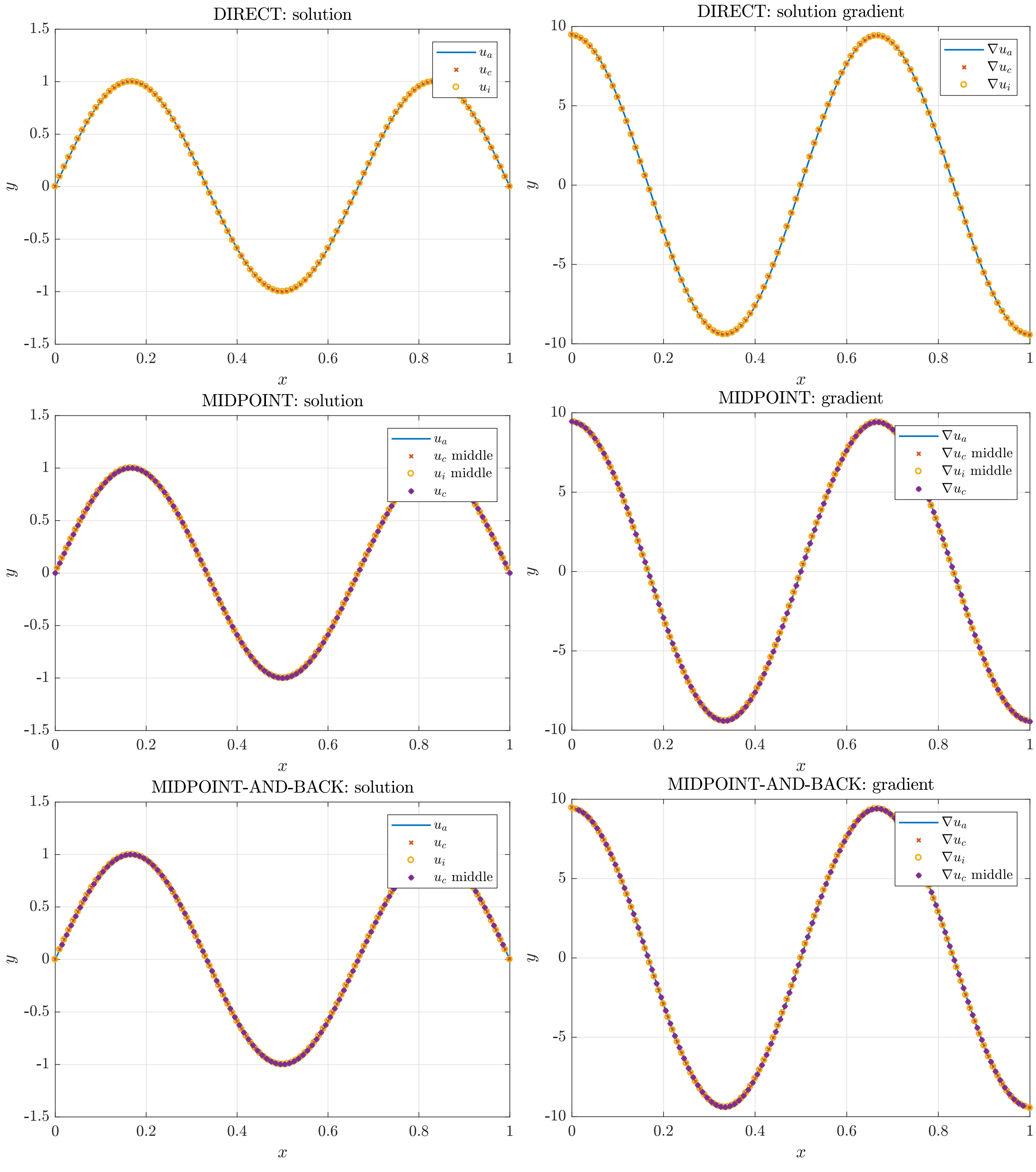

Figure 3: Solution behavior in 1D with $\boldsymbol{a} = (3)$, $n = 5$, $m = 2$ in DIRECT case and $n_s = 2$, $m_s = 1$, $n = 3$, $m = 2$ in other cases.

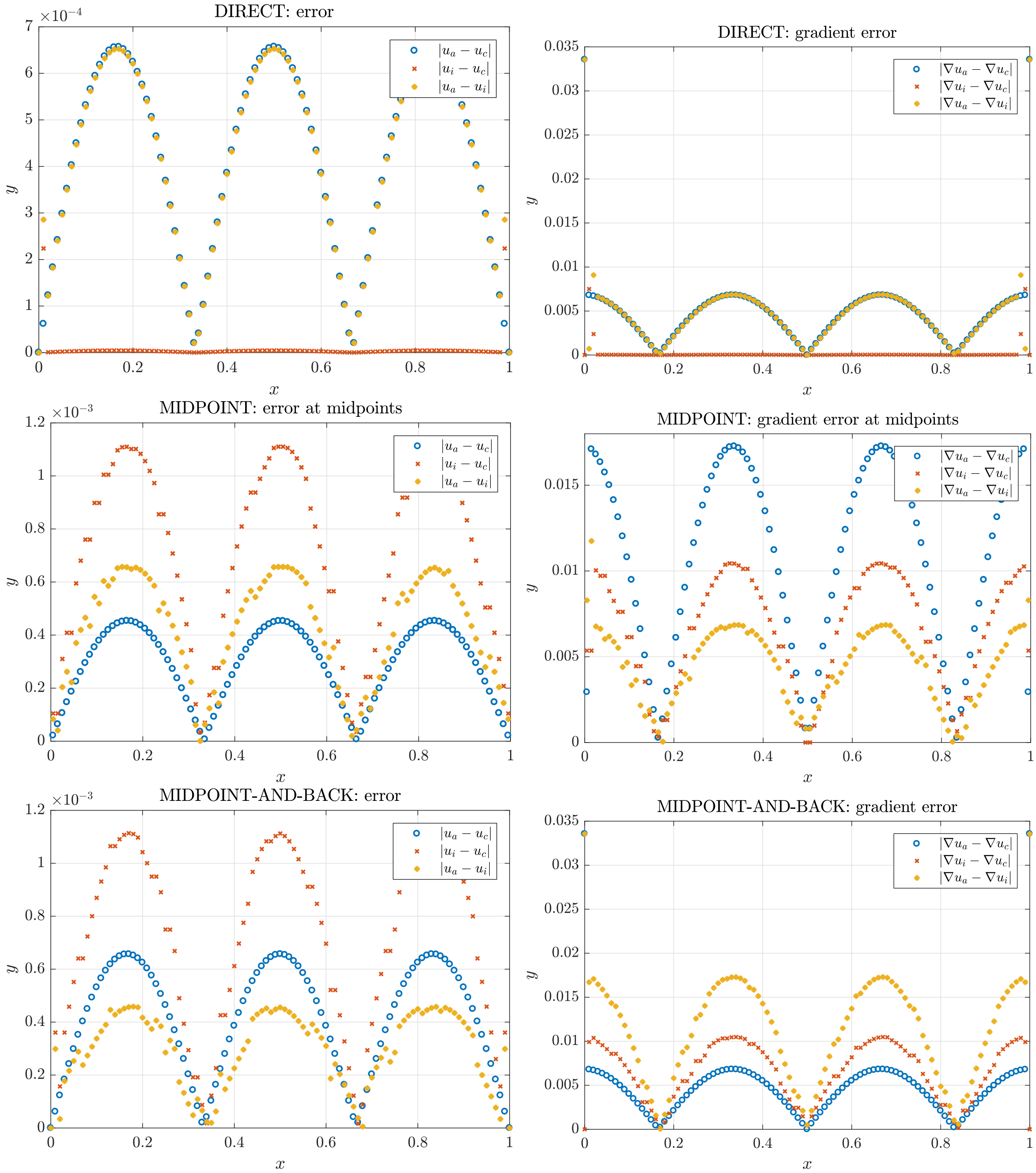

Figure 4: Indicator behavior in 1D with $\boldsymbol{a} = (3)$, $n = 5$, $m = 2$ in DIRECT case and $n_s = 2$, $m_s = 1$, $n = 3$, $m = 2$ in other cases.

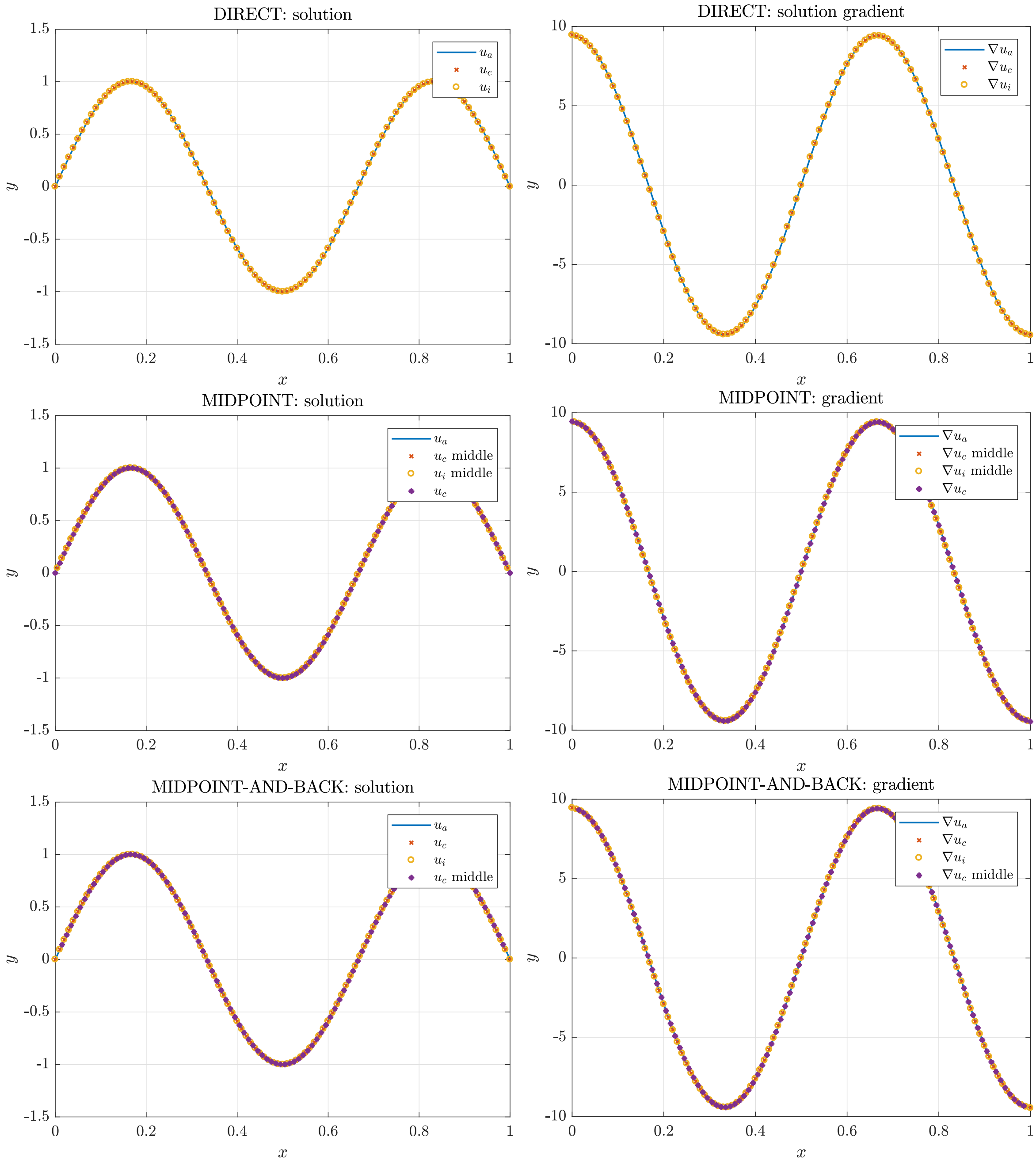

Figure 5: Solution behavior in 1D with $\boldsymbol{a} = (3)$, $n = 7$, $m = 3$ in DIRECT case and $n_s = 3$, $m_s = 2$, $n = 5$, $m = 3$ in other cases.

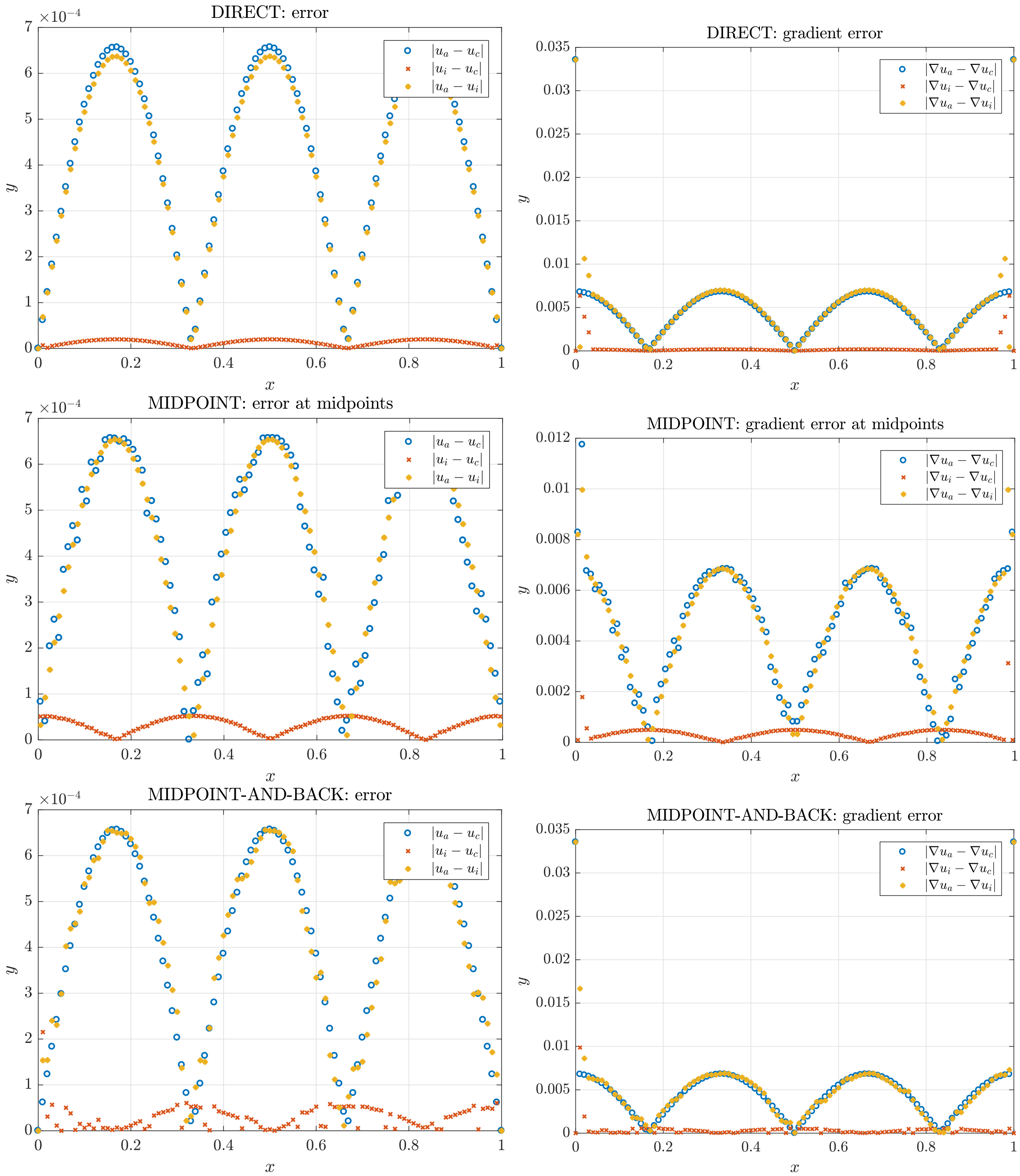

Figure 6: Indicator behavior in 1D with $\boldsymbol{a} = (3)$, $n = 7$, $m = 3$ in DIRECT case and $n_s = 3$, $m_s = 2$, $n = 5$, $m = 3$ in other cases.