Time and frequency analysis of non-uniform sampling

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Abstract - The spectrum of non-uniformly sampled signals is often needed for analysis of signals obtained from network of distributed sensors. In this paper, we analyze a method to retrieve the spectrum of this type of signals through interpolation, then re-sampling with uniform rate and performing Fourier transform at the end. We discuss this method based on the deviation of the non-uniform sampling rate and the interpolation type. Then, we examine the methodology on a non-uniformly sampled sine wave and a real ECG signal obtained from a Wireless Sensor Network.

I. INTRODUCTION

The main characteristic of distributed systems is that they have often similar, but independent clocks. Several mechanisms are known for the clock synchronization and alignment. Often, the algorithms for clock synchronization are quite complex and require a lot of communication and computing power [1]. Grids are an example of distributed systems where the synchronization between processes is implemented through the blocking communication [2, 3].

In this paper, we will focus on the wireless sensor networks (WSN) [4], which are a special case of distributed system in the sense that their clocks remain unsynchronized. We will consider the problem of data acquisition from the WSN nodes in a gateway node. Clock frequencies differ between the WSN nodes because of the demand for low power consumption [5], which forbids the use of clock synchronization protocols. Time stamping can be used instead, which works even if the power consumption is further lowered by keeping the communication unidirectional and only allowing data to flow from nodes towards the gateway [6].

Suppose that each WNS node acquires local data through local sensors, which operate with a local clock. Because of the inaccurate clocks, data cannot be sampled at the same time on all nodes. Even more, the clocks are drifting; therefore data sampling rate varies. Finally, because of unreliable radio channels, particularly if nodes are not stationary, a significant amount of data could be lost.

Data of this sort are collected at the gateway and often combined to obtain more complex information about the behavior of the whole system, not just about the local node. Combination of data is possible only if their sampling times are synchronized, which means that the acquired data samples must be aligned in time using corresponding time-stamps, interpolated and re-sampled with a common frequency.

The described problem could arise in many different contexts and on different scales. For example, in seismography where measured data are unsynchronized and non-uniformly sampled [7]; in geography the rainfall data from different places are also unsynchronized [8]; in radio astronomy where several spatially distributed receiving antennas constitute a single radio telescope [9], etc.

In this paper we have studied the data acquired form a body WSN (BWSN), where three bipolar wireless electrodes, representing three nodes, measure ECG potential locally and send them to the gateway [6]. The gateway combines these potentials in order to reconstruct the standard twelve channel ECG [6]. To reduce the power consumption for improved autonomy of the WSN, the communication must be implemented by protocols that enable extensive sleep mode of the transmitting nodes, thus data samples are packed in groups and send to the gateway in data packets together with node time-stamps. A schematic diagram of the BWSN architecture is shown in Fig.1.

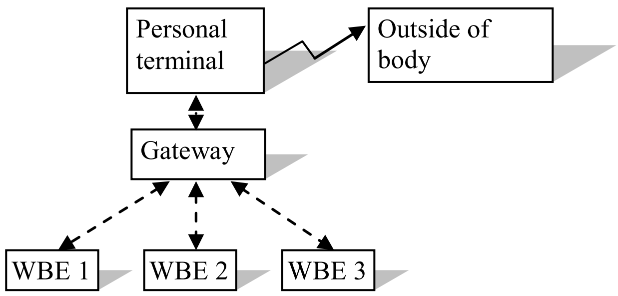

Fig. 1. A schematic diagram of the BWSN architecture. Three wireless body electrodes (WBEs) communicate with the gateway, which communicates with the outside world.

A personal terminal synchronizes the data from all the BWSN nodes to the same global time, by using gateway's and nodes' time-stamps to reconstruct sampling times. Using time-stamps from the last 20 received packets, the ratios between the gateway and node clock frequencies are maintained. Next, the signals are interpolated and re-sampled with a common frequency.

Using the history of time-stamps the gateway can also detect missing packets. The packets may get lost because of the simplicity of the communication protocol between the WBEs and the gateway. The retransmission of the packets is not feasible because of the simple one-way communication. Appropriate gaps of missing samples of the correct length are inserted into the data stream, based on the time-stamp analysis.

The synchronized, interpolated and re-sampled, data can now be further processed, e.g. standard 12-channel ECG can be synthesized [10].

The question remains to what extent the interpolation and re-sampling deteriorate the final results. We try to find the answer in the rest of this paper, which is organized as follows. In next section the methods for the analysis of errors in time and frequency domains are described. The results are shown in Section III. The paper concludes with discussion and future work.

II. METHOD

The analysis of errors introduced by interpolation and re-sampling is performed in time and frequency domain. The spectrum of a non-uniformly sampled signal is obtained by a three step method. First, non-uniform samples in time are interpolated, then a uniform re-sampling is applied and finally the Fast Fourier Transform (FFT) is performed to obtain the power spectrum.

We investigate three types of interpolation techniques: nearest neighbor, linear, and cubic spline. Nearest neighbor interpolation (NNI), also known as piecewise constant interpolation, is the simplest interpolation technique. For a non-given point, the nearest neighbor algorithm selects the value of the nearest point. Linear interpolation (LI) is implemented by a concatenation of linear interpolants between each pair of data points. For two known data points, the linear interpolant is the straight line between these points. Cubic spline interpolation (CSI) is a technique where the interpolant is a special type of piecewise cubic polynomial called a spline. Splines are chosen in such a way that they fit smoothly together.

After the interpolation, we re-sample the interpolated signal with uniform sampling rate, which is determined as the mean sampling rate of the initial non-uniformly sampled signal. Finally, the differences between the original signal and the signals obtained with the above methodology are analyzed both in time or frequency domain using several error measures. We calculate the Normalized Root Mean Square Error (NRMSE) in each domain, which is defined as RMSE divided by the range of observed values:

$$NRMSE = \frac{RMSE}{x_{\max} - x_{\min}} = \frac{\sqrt{\frac{\sum_{i=1}^{n}(x_i - y_i)^2}{n}}}{x_{\max} - x_{\min}} \quad (1)$$

where x_i and y_i are observed values, n is the number of analyzed signal points, and $(x_{max} - x_{min})$ is the signal range. We also calculate the normalized values of: Mean Absolute Error (NMAE), Maximal Absolute Error (NMaxAE), Minimal Absolute Error (NMinAE) and the Maximal Deviation from the Mean Absolute Error (NMaxDAE) to check whether there are some regions with big discrepancies. In all above acronyms N stands for Normalized – divided by the signal range.

We examine the proposed method on two non-uniformly sampled signals: sine wave and real ECG signal.

A. Non-uniformly sampled sine wave

We first generate a sine wave signal with frequency of 1 Hz and then uniformly sample it with sampling frequency of 20 Hz. We inspect 10 periods of the signal. In order to simulate non-uniformity in the sampling, we generate three new signals with different random deviation in the sampling rate (±0.1%, ±1% and ±10%). Then we apply all three interpolation techniques described previously. To visualize the non-uniformity in the sampling, a single period of non-uniformly sampled sine wave is shown in Fig.2.

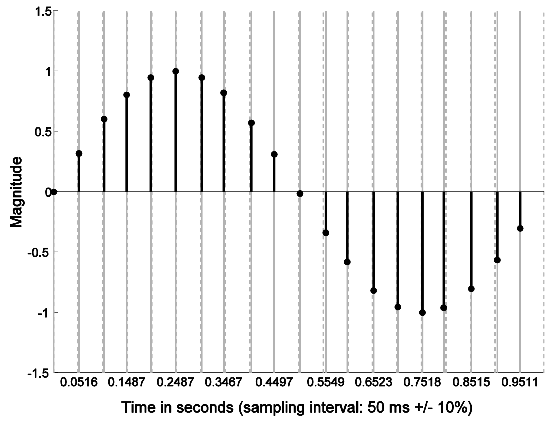

Fig. 2. Non-uniformly sampled sine wave with magnitude M = 1, phase φ = 0° and period T = 1 s.

B. Non-uniformly sampled ECG signal

We apply the same methodology also on a real ECG signal. A high resolution ECG signal with uniform sampling frequency of 1000 Hz is taken as a reference signal. Then the non-uniform down-sampling is applied with the average frequency of 100 Hz and a random deviation of ±10%. This signal could be regarded as an equivalent to the one obtained from a typical wireless ECG lead (WBE) that is a part of the BWSN, described in the previous section. We inspect 10 seconds of this signal. The errors are analyzed in the same way as in the case of the sine wave. The non-uniformly sampled ECG signal with sampling frequency 100 Hz ± 10%, interpolated with nearest neighbor technique, is shown in Fig.3.

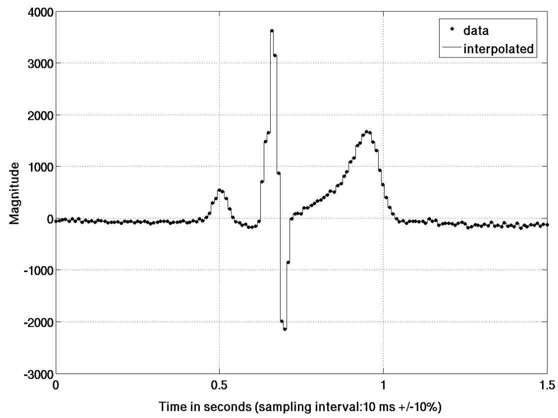

Fig. 3. Non-uniformly sampled ECG, interpolated by nearest neighbor technique.

III. RESULTS AND DISSCUSION

The results are obtained from error measures of 100 random generated non-uniform signals and presented in tables as mean value ± standard deviation.

A. Non-uniformly sampled sine wave

After re-sampling the interpolated non-uniformly sampled signals with a uniform sampling frequency, same as the one from the initial uniformly sampled sine wave (20 Hz), we calculate the NRMSE and other error measures between the initial uniformly sampled sine wave and the re-sampled one for all nine different settings, resulting from three sampling rate deviations (SRD) and three interpolation techniques. Results are listed in Table I. All three interpolation techniques show good performance - low NRMSE values. Nevertheless, the cubic spline interpolation performs the best - its NRMSE values are 3-4 orders of magnitude lower than the NRMSE values of the other interpolation techniques. The other error measures are also small, which indicates that there are no big discrepancies between the uniformly sampled and the re-sampled signal.

To see how the method reflects in the frequency domain, we perform FFT on each of the re-sampled signals and calculate the NRMSE and the other error measures between the power spectrum of the initial uniformly sampled sine wave and the power spectrum of the re-sampled signals. The power spectrum of a re-sampled signal and the normalized error from the power spectrum of the initial uniformly sampled sine wave are shown in Fig.4a and Fig.4b, respectively. Error analysis in frequency domain is given in Table II. The same conclusions apply in the frequency domain: all three interpolation techniques perform well, with the cubic spline interpolation being the best for the proposed method.

TABLE I
TIME DOMAIN ANALYSIS OF NON-UNIFORMLY SAMPLED SINE WAVE

SRD	Error Measures	NNI	LI	CSI
±0.1%	NRMSE	$6.38 \cdot 10^{-5} \pm 2.21 \cdot 10^{-6}$	$1.00 \cdot 10^{-5} \pm 4.01 \cdot 10^{-7}$	$5.66 \cdot 10^{-9} \pm 1.28 \cdot 10^{-9}$
	NMaxAE	$1.52 \cdot 10^{-4} \pm 3.73 \cdot 10^{-6}$	$2.37 \cdot 10^{-5} \pm 6.95 \cdot 10^{-7}$	$5.20 \cdot 10^{-8} \pm 1.95 \cdot 10^{-8}$
	NMinAE	$1.55 \cdot 10^{-10} \pm 3.98 \cdot 10^{-10}$	$4.45 \cdot 10^{-8} \pm 4.48 \cdot 10^{-8}$	$3.16 \cdot 10^{-12} \pm 4.13 \cdot 10^{-12}$
	NMAE	$4.92 \cdot 10^{-5} \pm 1.97 \cdot 10^{-6}$	$7.85 \cdot 10^{-6} \pm 3.45 \cdot 10^{-7}$	$3.11 \cdot 10^{-9} \pm 2.17 \cdot 10^{-10}$
	NMaxDAE	$1.02 \cdot 10^{-4} \pm 4.29 \cdot 10^{-6}$	$1.58 \cdot 10^{-5} \pm 7.03 \cdot 10^{-7}$	$4.89 \cdot 10^{-8} \pm 1.94 \cdot 10^{-8}$
±1.0%	NRMSE	$6.37 \cdot 10^{-4} \pm 2.57 \cdot 10^{-5}$	$1.00 \cdot 10^{-4} \pm 3.88 \cdot 10^{-6}$	$5.84 \cdot 10^{-8} \pm 1.08 \cdot 10^{-8}$
	NMaxAE	$1.51 \cdot 10^{-3} \pm 4.12 \cdot 10^{-5}$	$2.37 \cdot 10^{-4} \pm 6.23 \cdot 10^{-6}$	$5.24 \cdot 10^{-7} \pm 1.88 \cdot 10^{-7}$
	NMinAE	$1.30 \cdot 10^{-8} \pm 2.91 \cdot 10^{-8}$	$4.73 \cdot 10^{-7} \pm 4.57 \cdot 10^{-7}$	$8.53 \cdot 10^{-11} \pm 1.00 \cdot 10^{-12}$
	NMAE	$4.90 \cdot 10^{-4} \pm 2.12 \cdot 10^{-5}$	$7.81 \cdot 10^{-5} \pm 3.35 \cdot 10^{-6}$	$3.34 \cdot 10^{-8} \pm 2.28 \cdot 10^{-9}$
	NMaxDAE	$1.02 \cdot 10^{-3} \pm 4.19 \cdot 10^{-5}$	$1.59 \cdot 10^{-4} \pm 6.17 \cdot 10^{-6}$	$4.91 \cdot 10^{-7} \pm 1.87 \cdot 10^{-7}$
±10%	NRMSE	$6.34 \cdot 10^{-3} \pm 2.19 \cdot 10^{-4}$	$1.00 \cdot 10^{-3} \pm 3.59 \cdot 10^{-5}$	$1.69 \cdot 10^{-6} \pm 1.38 \cdot 10^{-7}$
	NMaxAE	$1.51 \cdot 10^{-2} \pm 3.91 \cdot 10^{-4}$	$2.45 \cdot 10^{-3} \pm 1.06 \cdot 10^{-4}$	$6.62 \cdot 10^{-6} \pm 1.19 \cdot 10^{-6}$
	NMinAE	$1.06 \cdot 10^{-6} \pm 2.03 \cdot 10^{-6}$	$5.02 \cdot 10^{-6} \pm 4.83 \cdot 10^{-6}$	$1.32 \cdot 10^{-9} \pm 1.24 \cdot 10^{-9}$
	NMAE	$4.88 \cdot 10^{-3} \pm 2.06 \cdot 10^{-4}$	$7.76 \cdot 10^{-4} \pm 3.10 \cdot 10^{-5}$	$1.10 \cdot 10^{-6} \pm 8.68 \cdot 10^{-8}$
	NMaxDAE	$1.02 \cdot 10^{-2} \pm 4.09 \cdot 10^{-4}$	$1.67 \cdot 10^{-3} \pm 1.07 \cdot 10^{-4}$	$5.52 \cdot 10^{-6} \pm 1.16 \cdot 10^{-6}$

TABLE II
FREQUENCY DOMAIN ANALYSIS OF NON-UNIFORMLY SAMPLED SINE WAVE

SRD	Error Measures	NNI	LI	CSI
±0.1%	NRMSE	$1.36 \cdot 10^{-5} \pm 1.00 \cdot 10^{-6}$	$2.80 \cdot 10^{-6} \pm 1.24 \cdot 10^{-7}$	$1.15 \cdot 10^{-9} \pm 4.02 \cdot 10^{-10}$
	NMaxAE	$3.87 \cdot 10^{-5} \pm 5.82 \cdot 10^{-6}$	$2.46 \cdot 10^{-5} \pm 1.22 \cdot 10^{-6}$	$3.07 \cdot 10^{-9} \pm 7.07 \cdot 10^{-10}$
	NMinAE	$1.27 \cdot 10^{-7} \pm 1.22 \cdot 10^{-7}$	$1.22 \cdot 10^{-8} \pm 1.39 \cdot 10^{-8}$	$1.36 \cdot 10^{-11} \pm 1.62 \cdot 10^{-11}$
	NMAE	$1.09 \cdot 10^{-5} \pm 8.29 \cdot 10^{-7}$	$1.31 \cdot 10^{-6} \pm 7.73 \cdot 10^{-8}$	$9.47 \cdot 10^{-10} \pm 3.69 \cdot 10^{-10}$
	NMaxDAE	$2.78 \cdot 10^{-5} \pm 5.58 \cdot 10^{-6}$	$2.33 \cdot 10^{-5} \pm 1.18 \cdot 10^{-6}$	$2.13 \cdot 10^{-9} \pm 5.16 \cdot 10^{-10}$
±1.0%	NRMSE	$1.33 \cdot 10^{-4} \pm 1.06 \cdot 10^{-5}$	$2.78 \cdot 10^{-5} \pm 1.18 \cdot 10^{-6}$	$1.24 \cdot 10^{-8} \pm 3.71 \cdot 10^{-9}$
	NMaxAE	$3.76 \cdot 10^{-4} \pm 4.73 \cdot 10^{-5}$	$2.44 \cdot 10^{-4} \pm 1.22 \cdot 10^{-5}$	$3.57 \cdot 10^{-8} \pm 6.45 \cdot 10^{-9}$
	NMinAE	$1.28 \cdot 10^{-6} \pm 1.33 \cdot 10^{-6}$	$1.39 \cdot 10^{-7} \pm 1.29 \cdot 10^{-7}$	$1.37 \cdot 10^{-10} \pm 1.37 \cdot 10^{-10}$
	NMAE	$1.08 \cdot 10^{-4} \pm 8.80 \cdot 10^{-6}$	$1.30 \cdot 10^{-5} \pm 6.66 \cdot 10^{-7}$	$1.01 \cdot 10^{-8} \pm 3.48 \cdot 10^{-9}$
	NMaxDAE	$2.68 \cdot 10^{-4} \pm 4.43 \cdot 10^{-5}$	$2.31 \cdot 10^{-4} \pm 1.20 \cdot 10^{-5}$	$2.55 \cdot 10^{-8} \pm 5.00 \cdot 10^{-9}$
±10%	NRMSE	$1.35 \cdot 10^{-3} \pm 7.15 \cdot 10^{-5}$	$2.78 \cdot 10^{-4} \pm 1.13 \cdot 10^{-5}$	$4.27 \cdot 10^{-7} \pm 3.87 \cdot 10^{-8}$
	NMaxAE	$3.88 \cdot 10^{-3} \pm 5.30 \cdot 10^{-4}$	$2.43 \cdot 10^{-3} \pm 1.12 \cdot 10^{-4}$	$3.07 \cdot 10^{-6} \pm 2.97 \cdot 10^{-7}$
	NMinAE	$1.24 \cdot 10^{-5} \pm 1.27 \cdot 10^{-5}$	$1.26 \cdot 10^{-6} \pm 1.29 \cdot 10^{-6}$	$2.30 \cdot 10^{-9} \pm 2.15 \cdot 10^{-9}$
	NMAE	$1.07 \cdot 10^{-3} \pm 6.18 \cdot 10^{-5}$	$1.31 \cdot 10^{-4} \pm 7.43 \cdot 10^{-6}$	$2.68 \cdot 10^{-7} \pm 2.72 \cdot 10^{-8}$
	NMaxDAE	$2.81 \cdot 10^{-3} \pm 5.30 \cdot 10^{-4}$	$2.30 \cdot 10^{-3} \pm 1.08 \cdot 10^{-4}$	$2.81 \cdot 10^{-6} \pm 2.80 \cdot 10^{-7}$

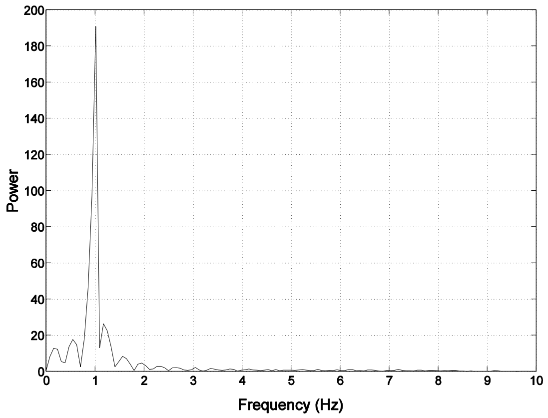

Fig. 4a. Power spectrum of re-sampled NNI non-uniformly sampled sine wave (sampling rate deviation ±10%).

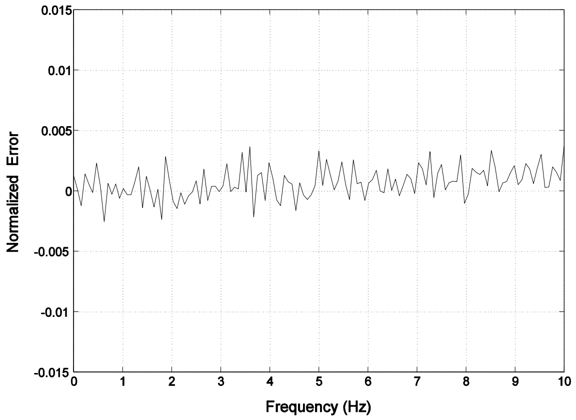

Fig. 4b. Normalized error of the power spectrum from Fig. 4a.

B. Non-uniformly sampled ECG signal

The same error analysis is applied also on both ECG signals, sampled uniformly and non-uniformly. The errors in time domain are listed in Table III and in frequency domain in Table IV. The power spectrum of the re-sampled NNI non-uniformly sampled ECG signal and the normalized error from the power spectrum of the uniformly sampled ECG signal are shown in Fig.5a and Fig.5b, respectively.

TABLE III
TIME DOMAIN ANALYSIS OF NON-UNIFORMLY SAMPLED ECG SIGNAL

SRD	Error Measures	NNI	LI	CSI
±10%	NRMSE	$6.04 \cdot 10^{-3}$ $\pm 3.29 \cdot 10^{-4}$	$4.41 \cdot 10^{-3}$ $\pm 2.62 \cdot 10^{-4}$	$3.37 \cdot 10^{-3}$ $\pm 2.15 \cdot 10^{-4}$
	NMaxAE	$6.08 \cdot 10^{-2}$ $\pm 5.17 \cdot 10^{-3}$	$4.00 \cdot 10^{-2}$ $\pm 4.17 \cdot 10^{-3}$	$4.02 \cdot 10^{-2}$ $\pm 4.75 \cdot 10^{-3}$
	NMinAE	0 ± 0	0 ± 0	0 ± 0
	NMAE	$2.46 \cdot 10^{-3}$ $\pm 1.12 \cdot 10^{-4}$	$2.03 \cdot 10^{-3}$ $\pm 8.72 \cdot 10^{-5}$	$1.74 \cdot 10^{-3}$ $\pm 7.47 \cdot 10^{-5}$
	NMaxDAE	$5.84 \cdot 10^{-2}$ $\pm 5.14 \cdot 10^{-3}$	$3.80 \cdot 10^{-2}$ $\pm 4.16 \cdot 10^{-3}$	$3.85 \cdot 10^{-2}$ $\pm 4.73 \cdot 10^{-3}$

TABLE IV
FREQUENCY DOMAIN ANALYSIS OF NON-UNIFORMLY SAMPLED ECG SIGNAL

SRD	Error Measures	NNI	LI	CSI
±10%	NRMSE	$4.24 \cdot 10^{-3}$ $\pm 2.73 \cdot 10^{-4}$	$3.55 \cdot 10^{-3}$ $\pm 2.71 \cdot 10^{-4}$	$2.38 \cdot 10^{-3}$ $\pm 1.73 \cdot 10^{-4}$
	NMaxAE	$1.33 \cdot 10^{-2}$ $\pm 1.64 \cdot 10^{-3}$	$1.15 \cdot 10^{-2}$ $\pm 1.49 \cdot 10^{-3}$	$7.58 \cdot 10^{-3}$ $\pm 8.95 \cdot 10^{-4}$
	NMinAE	$1.09 \cdot 10^{-5}$ $\pm 1.00 \cdot 10^{-5}$	$1.04 \cdot 10^{-5}$ $\pm 9.76 \cdot 10^{-6}$	$5.26 \cdot 10^{-6}$ $\pm 4.70 \cdot 10^{-6}$
	NMAE	$3.39 \cdot 10^{-3}$ $\pm 2.26 \cdot 10^{-4}$	$2.82 \cdot 10^{-3}$ $\pm 2.11 \cdot 10^{-4}$	$1.90 \cdot 10^{-3}$ $\pm 1.36 \cdot 10^{-4}$
	NMaxDAE	$9.92 \cdot 10^{-3}$ $\pm 1.59 \cdot 10^{-3}$	$8.64 \cdot 10^{-3}$ $\pm 1.38 \cdot 10^{-3}$	$5.68 \cdot 10^{-3}$ $\pm 8.42 \cdot 10^{-4}$

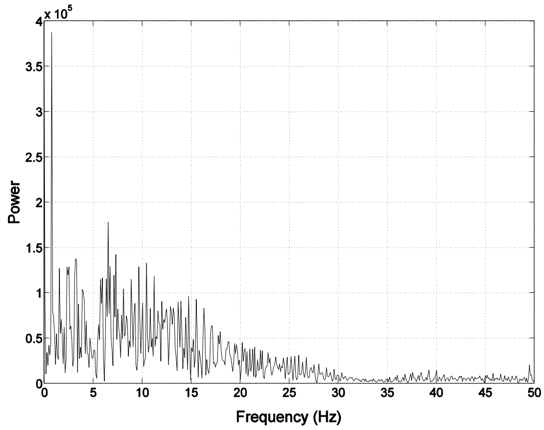

Fig. 5a. Power spectrum of re-sampled NNI non-uniformly sampled ECG signal.

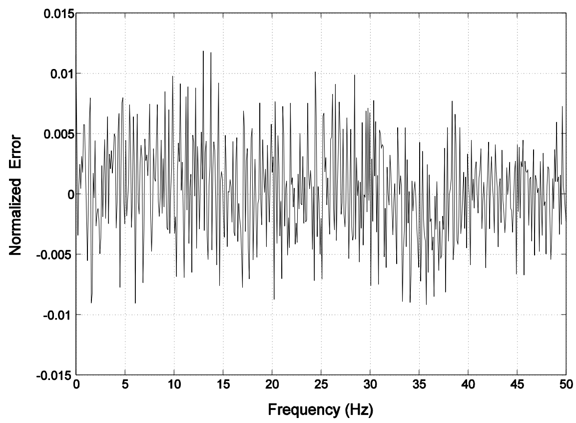

Fig. 5b. Normalized error of the power spectrum from Fig. 5a.

The results follow the same trend as for the sine wave signal. Here also, the three interpolation techniques introduce low NRMSE and other error values with the cubic spline interpolation performing the best. On the other hand, error analysis of ECG no longer shows orders of magnitude better performance by the CSI; all interpolation techniques come quite close.

IV. CONCLUSION

The obtained results indicate that the influence of different interpolation techniques applied before re-sampling is small. The errors in both domains grow with the amount of deviation in the sampling frequency. The interpolation technique with the smallest error is cubic spline. However, it is also the most computationally complex. The relations between errors in time and frequency domain remain the same; therefore any domain can be analyzed. As future work the impact of missing data packets should be analyzed, regarding the method for their reconstruction and induced errors as a function of the amount of missing data.

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